Designation: E 808 – 01

Standard Practice for
Describing Retroreflection[1]

This standard is issued under the fixed designation E 808; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice provides terminology, alternative geometrical coordinate systems, and procedures for designating angles in descriptions of retroreflectors, specifications for retroreflector performance, and measurements of retroreflection.

1.2 Terminology defined herein includes terms germane to other ASTM documents on retroreflection.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*
E 284 Terminology of Appearance[2]

2.2 *Federal Standard:*
Fed. Std. No. 370 Instrumental Photometric Measurements of Retroreflecting Materials and Retroreflecting Devices[3]

2.3 *CIE Document:*
CIE Publication No. 54, Retroreflection-Definition and Measurement[4]

3. Terminology

3.1 Terms and definitions in Terminology E 284 are applicable to this standard.

3.1.1 In accordance with the convention appearing in the Significance and Use section of Terminology E 284, the superscript *B* appearing after [CIE] at the end of a definition indicates that the given definition is a modification of that cited with little difference in essential meaning.

NOTE 1—The terminology given here describes visual observation of luminance as defined by the CIE V (λ) spectral weighting function for the photopic observer. Analogous terms for other purposes can be defined by using appropriate spectral weighting.

3.2 *Definitions: The delimiting phrase "in retroreflection" applies to each of the following definitions when used outside the context of this or other retroreflection standards.*

3.2.1 *coefficient of line retroreflection,* R_M, *n*—of a retroreflecting stripe, the ratio of the coefficient of luminous intensity (R_I) to the length (l), expressed in candelas per lux per metre ($cd \cdot lx^{-1} \cdot m^{-1}$). $R_M = R_I/l$.

3.2.1.1 *Discussion*—R_M depends on the spectral composition of the illumination which is usually CIE illuminant A.

3.2.2 *coefficient of luminous intensity,* R_I, *n*—of a retroreflector, ratio of the luminous intensity (I) of the retroreflector in the direction of observation to the illuminance ($E_\perp$) at the retroreflector on a plane perpendicular to the direction of the incident light, expressed in candelas per lux ($cd \cdot lx^{-1}$). $R_I = (I/E_\perp)$.

3.2.2.1 *Discussion*—In a given measurement one obtains the average R_I over the solid angles of incidence and viewing subtended by the source and receiver apertures, respectively. In practice, I is often determined as the product of the illuminance at the observer and the distance squared ($I = E_r d^2$). R_I depends on the spectral composition of the illumination which is usually CIE illuminant A.

3.2.2.2 *Discussion*—Also called *coefficient of (retroreflected) luminous intensity.* Equivalent commonly used terms are *CIL* and *SI* (specific intensity). CIE Publication 54 uses the symbol *R* for R_I. The ASTM recommendation is to use the symbol R_I.

3.2.3 *coefficient of retroreflected luminance,* R_L, *n*—the ratio of the luminance, *L*, in the direction of observation to the normal illuminance, $E_\perp$, at the surface on a plane normal to the incident light, expressed in candelas per square metre per lux [$(cd \cdot m^{-2}) \cdot lx^{-1}$].

$$R_L = (L/E_\perp) = (R_I/A \cos v) = (I/EA \cos v) = (R_A/ \cos v) \qquad (1)$$

where:
A = surface area of the sample, and
v = viewing angle.

3.2.3.1 *Discussion*—The units millicandela per square metre per lux [$(mcd \cdot m^{-2}) \cdot lx^{-1}$] are usually used to express the R_L values of road marking surfaces. This quantity is also referred to as specific luminance. Historically the symbol *SL* was used for R_L. In some references *CRL* is used. These are all

equivalent, but R_L is preferred.

3.2.3.2 *Discussion*—R_L depends on the spectral composition of the illumination which is usually CIE illuminant A.

3.2.4 *coefficient of (retroreflected) luminous flux, R_Φ, n*—the ratio of the luminous flux per unit solid angle, Φ'/Ω', in the direction of observation to the total flux Φ incident on the effective retroreflective surface, expressed in candelas per lumen (cd·lm^{-1}).

$$R_\Phi = (\Phi'/\Omega')/\Phi = I/\Phi = R_A/\cos\ \beta \qquad (2)$$

3.2.4.1 *Discussion*—The units for this photometric quantity, candelas per lumen, are sometimes abbreviated as *CPL*.

3.2.4.2 *Discussion*—R_Φ depends on the spectral composition of the illumination which is usually CIE illuminant A.

3.2.5 *coefficient of retroreflection, R_A, n*—of a plane retroreflecting surface, the ratio of the coefficient of luminous intensity (R_I) to the area (A), expressed in candelas per lux per square metre (cd·lx^{-1}· m^{-2}). $R_A = R_I/A$.

3.2.5.1 *Discussion*—The equivalent inch-pound units for coefficient of retroreflection are candelas per foot candle per square foot. The SI and inch-pound units are numerically equal, because the units of R_A reduce to 1/sr. An equivalent term used for coefficient of retroreflection is specific intensity per unit area, with symbol SIA or the CIE symbol R'. The term coefficient of retroreflection and the symbol R_A along with the SI units of candelas per lux per square metre are recommended by ASTM.

3.2.5.2 *Discussion*—The radiometric BRDF is not the analogue of R_A but rather of R_Φ.

3.2.5.3 *Discussion*—R_A depends on the spectral composition of the illumination which is usually CIE illuminant A.

3.2.6 *co-entrance angle, e, n*—the complement of the angle between the retroreflector axis and the illumination axis.

3.2.6.1 *Discussion*—$e=90°-\beta$. Range $0°<e≤90°$. For horizontal road markings, the retroreflector axis is considered to be the normal to the road surface, making e the angle of inclination of the illumination axis over the road surface.

3.2.7 *co-viewing angle, a, n*—the complement of the angle between the retroreflector axis and the observation axis.

3.2.7.1 *Discussion*—$a= 90°-v$. Range $0°<a≤90°$. For horizontal road markings, the retroreflector axis is considered to be the normal to the road surface, making a the angle of inclination of the observation axis over the road surface.

3.2.8 *datum axis, n*—a designated half-line from the retroreflector center perpendicular to the retroreflector axis.

3.2.8.1 *Discussion*—The datum axis together with the retroreflector center and the retroreflector axis establish the position of the retroreflector.

3.2.9 *datum mark, n*—an indication on the retroreflector, off the retroreflector axis, that establishes the direction of the datum axis.

3.2.10 *datum half-plane, n*—the half-plane that originates on the line of the retroreflector axis and contains the datum axis.

3.2.11 *entrance angle, β, n*—the angle between the illumination axis and the retroreflector axis.

3.2.11.1 *Discussion*—The entrance angle is usually no larger than 90°, but for completeness its full range is defined as $0°≤\beta≤180°$. In the CIE (goniometer) system β is resolved into

two components β_1 and β_2. Since by definition β is always positive, the common practice of referring to the small entrance angles that direct specular reflections away from the photoreceptor as negative valued is deprecated by ASTM. The recommendation is to designate such negative values as belonging to β_1.

3.2.12 *entrance angle component, β_1, n*—the angle from the illumination axis to the plane containing the retroreflector axis and the first axis. Range: $-180°<\beta_1≤180°$.

3.2.13 *entrance angle component, β_2, n*—the angle from the plane containing the observation half-plane to the retroreflector axis. Range: $-90°≤\beta_2≤90°$.

3.2.13.1 *Discussion*—For some measurements it is convenient to extend the range of β_2 to $-180°<\beta_2≤180°$. β_1 must then be restricted to $-90°<\beta_1≤90°$.

3.2.14 *entrance half-plane, n*—the half-plane that originates on the line of the illumination axis and contains the retroreflector axis.

3.2.15 *first axis, n*—the axis through the retroreflector center and perpendicular to the observation half-plane.

3.2.16 *fractional retroreflectance, R_T, n*—the fraction of unidirectional flux illuminating a retroreflector that is received at observation angles less than a designated value, α_{max}.

3.2.16.1 *Discussion*—R_T has no meaning unless α_{max} is specified.

3.2.16.2 *Discussion*—For a flat retroreflector R_T may be calculated as follows:

$$\int_{\alpha=0}^{\alpha_{max}} \int_{\rho=-\pi}^{\pi} \alpha\, \frac{R_A(\alpha,\rho)}{\cos\beta}\, d\alpha d\rho. \qquad (3)$$

For a non-flat retroreflector R_T may be calculated as follows:

$$\int_{\alpha=0}^{\alpha_{max}} \int_{\rho=-\pi}^{\pi} \alpha\, \frac{R_I(\alpha,\rho)}{A_P}\, d\alpha d\rho. \qquad (4)$$

A_P is the area of the retroreflector as projected in the direction of illumination. Angles β and ω_s must remain fixed through the integration. Angles α and ρ are in radians: R_T is unitless. Presentation angle γ may replace ρ in these formulas. For very small values of β, rotation angle ϵ may replace ρ in these formulas. For example, for $\beta=5°$ the resulting error will be less than, usually much less than, 0.5 % of the calculated R_T.

3.2.16.3 *Discussion*—R_T is usually expressed in percent.

3.2.17 *illumination axis, n*—the half-line from the retroreflector center through the source point.

3.2.18 *illumination distance, n*—the distance between the source point and the retroreflector center.

3.2.19 *observation angle, α, n*—the angle between the illumination axis and the observation axis.

3.2.19.1 *Discussion*—The observation angle is never negative and is almost always less than 10° and usually no more than 2°. The full range is defined as $0°≤\alpha<180°$.

3.2.20 *observation axis, n*—the half-line from the retroreflector center through the observation point.

3.2.21 *observation distance, d, n*—the distance between the retroreflector center and the observation point.

3.2.22 *observation half-plane, n*—the half-plane that originates on the line of the illumination axis and contains the observation axis.

3.2.23 *observation point, n*—the point taken as the location of the receiver.

3.2.23.1 *Discussion*—in real systems the receiver has finite

size and the observation point is typically the center of the entrance pupil.

3.2.24 *orientation angle,* ω_s, *n*—the angle in a plane perpendicular to the retroreflector axis from the entrance half-plane to the datum axis, measured counter-clockwise from the viewpoint of the source.

3.2.24.1 *Discussion*—Range $-180° < \omega_s \leq 180°$. In the previous editions of Practice E 808 as well as in CIE Pub. 54, 1982, orientation angle is defined as ω, the supplement of the above defined orientation angle ω_s. The change reverses the sense of orientation angle, making it now agree with the counterclockwise sense of rotation angle, ϵ, and exchanges the 0° and 180° points, making it now agree with Fed. Std. No. 370, §2.2.9b.

3.2.25 *presentation angle,* γ, *n*—the dihedral angle from the entrance half-plane to the observation half-plane, measured counter-clockwise from the viewpoint of the source.

3.2.25.1 *Discussion*—Range $-180° < \gamma \leq 180°$.

3.2.26 *retroreflectance factor,* R_F, (of a plane retroreflecting surface), *n*—the dimensionless ratio of the coefficient of luminous intensity (R_I) of a plane retroreflecting surface having area A to the coefficient of luminous intensity of a perfect reflecting diffuser of the same area under the same conditions of illumination and observation.

$$R_F = \frac{\pi R_I}{A \cos \beta \cos \nu} \qquad (5)$$

3.2.26.1 *Discussion*—In the above expression β is the entrance angle and ν is the viewing angle. The quantity, R_F, is numerically the same as the reflectance factor, R.

3.2.26.2 *Discussion*—R_F depends on the spectral composition of the illumination which is usually CIE illuminant A.

3.2.27 *retroreflection,* *n*—reflection in which reflected rays are preferentially returned in directions close to the opposite of the direction of the incident rays, this property being maintained over wide variations of the direction of the incident rays. [CIE] [B]

3.2.28 *retroreflective device,* *n*—deprecated term; use *retroreflector.*

3.2.29 *retroreflective element,* *n*—a minimal optical unit that produces retroreflection.

3.2.30 *retroreflective material,* *n*—a material that has a thin continuous layer of small retroreflective elements on or very near its exposed surface (for example, retroreflective sheeting, beaded paint, highway sign surfaces, or pavement striping).

3.2.31 *retroreflective sheeting,* *n*—a retroreflective material preassembled as a thin film ready for use.

3.2.32 *retroreflector,* *n*—a reflecting surface or device from which, when directionally irradiated, the reflected rays are preferentially returned in directions close to the opposite of the direction of the incident rays, this property being maintained over wide variations of the direction of the incident rays. [CIE][B]

3.2.33 *retroreflector axis,* *n*—a designated half-line from the retroreflector center.

3.2.33.1 *Discussion*—The direction of the retroreflector axis is usually chosen centrally among the intended directions of illumination; for example, the direction of the road on which or with respect to which the retroreflector is intended to be positioned. When symmetry exists, the retroreflector axis usually coincides with the axis of symmetry of the retroreflector. For horizontal road markings the normal to the surface is chosen as the retroreflector axis.

3.2.34 *retroreflector center,* *n*—the point on or near a retroreflector that is designated to be the location of the device.

3.2.35 *rho angle,* ρ, *n*—the dihedral angle from the observation half-plane to the half-plane that originates on the line of the illumination axis and contains the datum axis, measured counter-clockwise from the viewpoint of the source.

3.2.35.1 *Discussion*—Range $-180° < \rho \leq 180°$.

3.2.36 *RM azimuthal angle,* b, *n*—the dihedral angle from the half-plane originating on the line of the retroreflector axis and containing the obverse of the illumination axis to the half-plane originating on the line of the retroreflector axis and containing the observation axis, measured clockwise from a viewpoint on the retroreflector axis.

3.2.36.1 *Discussion*—Range $-180° < b \leq 180°$.

3.2.37 *RM supplemental azimuthal angle,* d, *n*— the angle in a plane perpendicular to the retroreflector axis from the obverse of the datum axis to the half-plane that originates on the line of the retroreflector axis and contains the observation axis, measured clockwise from a viewpoint on the retroreflector axis.

3.2.37.1 *Discussion*—Range $-180° < d \leq 180°$.

3.2.38 *rotation angle,* ϵ, *n*—the angle in a plane perpendicular to the retroreflector axis from the observation half-plane to the datum axis, measured counter-clockwise from a viewpoint on the retroreflector axis.

3.2.38.1 *Discussion*—Range $-180° < \epsilon \leq 180°$. The definition is applicable when entrance angle and viewing angle are less than 90°. More generally, rotation angle is the angle from the positive part of second axis to the datum axis, measured counterclockwise from a viewpoint on the retroreflector axis.

3.2.38.2 *Discussion*—Rotation of the sample about the retroreflector axis while the source and receiver remain fixed in space changes the rotation angle (ϵ) and the orientation angle (ω_s) equally.

3.2.39 *rotationally uniform,* *adj*—having substantially constant R_A, R_I, or R_L when rotated about the retroreflector axis, while the source, receiver, retroreflector center and retroreflector axis all remain in a fixed spatial relation.

3.2.39.1 *Discussion*—The orientation angle (ω_s) and the rotation angle (ϵ) both vary through 360° as the retroreflector is rotated about its axis, while the observation angle, entrance angle (both components β_1 and β_2) and presentation angle (γ) remain constant. A retroreflector may be rotationally uniform for some values of the constant angles and not for others.

3.2.39.2 *Discussion*—The degree of rotational uniformity can be specified numerically.

3.2.40 *second axis,* *n*—the axis through the retroreflector center, lying in the plane of the illumination axis and observation axis and perpendicular to the retroreflector axis; its positive direction lies in the observation half plane when $-90° \leq \beta_1 \leq 90°$ as shown in Fig. 1.

3.2.40.1 *Discussion*—The second axis is perpendicular also to the first axis.

3.2.41 *source point,* *n*—the point taken as the location of the source of illumination.
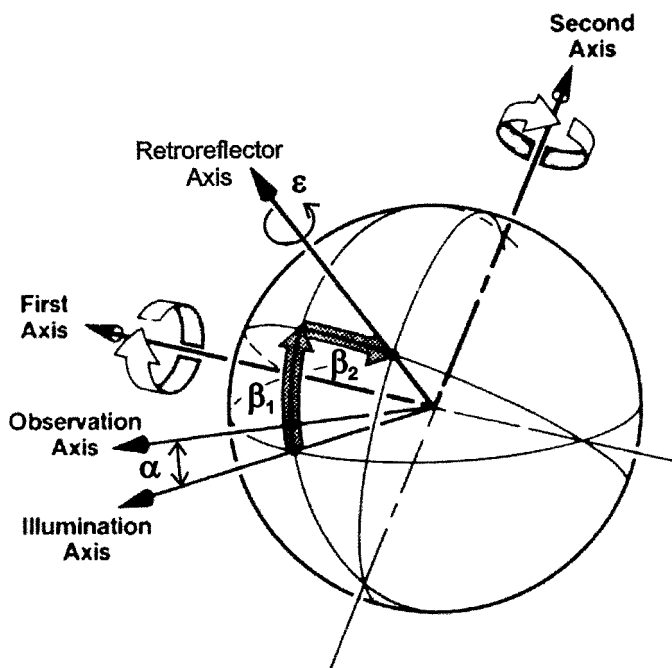
NOTE 1—The first axis is perpendicular to the plane containing the observation axis and the illumination axis. The second axis is perpendicular to the retroreflector axis and lies in the plane containing the observation axis and the illumination axis. All axes, angles, and directions are shown positive.

FIG. 1 CIE (Goniometer) System for Measuring Retroreflectors

3.2.41.1 *Discussion*—In real systems, the light source has finite size, and the source point is typically the center of the exit pupil.

3.2.42 *viewing angle, v, n*—the angle between the retroreflector axis and the observation axis.

3.2.42.1 *Discussion*—In the CIE (goniometer) system $\cos v = \cos(\beta_1 - \alpha)\cos\beta_2$. When the viewing angle is near 90°, as is normally the case for horizontal road markings, it may be convenient to use the co-viewing angle a, the complement of the viewing angle.

3.3 *Abbreviations:Abbreviated Form:*

3.3.1 *RM*—Road Marking.

4. Significance and Use

4.1 This practice applies to any measurement of reflectance in which the angle at the sample between the direction of the incident radiation and the direction of viewing is less than approximately 10°, and the reflected radiation is concentrated in a direction opposite to the direction of incidence.

4.2 The CIE (goniometer) system described in 6.1.1 was developed by the Subcommittee on Retroreflection of Committee 2.3 on Materials of the International Commission on Illumination (Commission International de l'Eclairage, CIE). It is intended to provide a common basis for the measurement of retroreflection, which should be used worldwide.

4.3 This practice provides alternative geometric coordinate systems useful for visualizing relationships between various angles in actual use.

5. Describing Retroreflection

5.1 *Specimen Geometry*—Designating the retroreflector center, the retroreflector axis and the datum axis establishes

coordinates fixed with respect to the retroreflector by means of which its location and angular orientation can be specified. For a retroreflector, these geometric parameters are usually related to the intended use, and are designated by the manufacturer.

5.2 *Illumination Geometry*—Entrance angle β and orientation angle ω_s together completely specify the illumination of the sample.

5.3 *Observation Geometry*—Observation angle α and rho angle ρ together completely specify the observation of the sample.

NOTE 2—Rotation angle ϵ is the projection of ρ into a plane perpendicular to the retroreflector axis, so α and ϵ together can also specify the observation of the sample with respect to the datum axis.

NOTE 3—Presentation angle γ lies in the plane of ρ, and when the receiver moves while the sample and the source are fixed in space, the change in γ is equal to the change in ρ. Thus α and γ together specify the observation of the sample with respect to the illumination of the sample, but not with respect to the datum axis of the sample. Fig. 2 shows the interrelation of angles $\alpha,\beta,\epsilon,\omega_s,\gamma,\rho$.

6. Geometrical Systems

6.1 A combination of four angles is necessary and sufficient for describing the geometric relations of source, receiver, and sample. Several systems have been devised using angles defined in Section 5. The four angles in any system can be calculated from the four angles in any other. Transformations are given in Appendix X1. The following four systems are noteworthy:

(*1*) $\{\alpha,\beta_1,\beta_2,\epsilon\}$ CIE (goniometer) system,

(*2*) $\{\alpha,\beta,\gamma,\omega_s\}$ Intrinsic system,

(*3*) $\{\alpha,\beta,\epsilon,\omega_s\}$ Application system, and

(*4*) $\{a,b,e,d\}$ RM system (Road Marking system).

The first three systems are spherical; illumination and
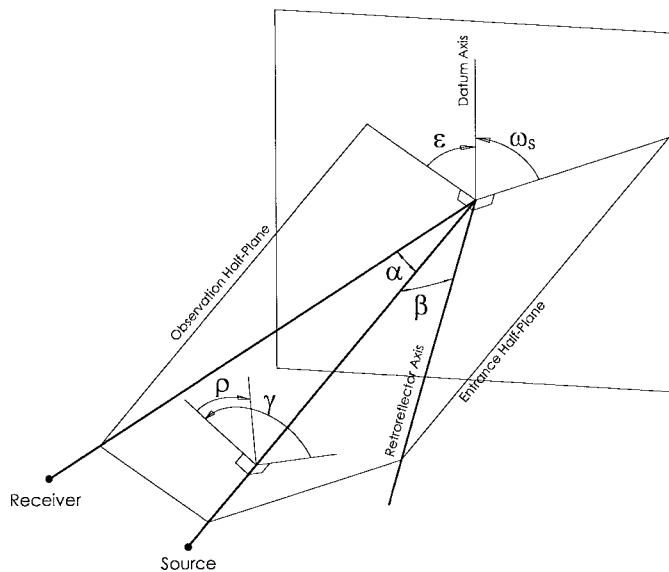
NOTE 1—Angles ϵ and ρ have clockwise senses in this figure, so their values are negative.
FIG. 2 Interrelation of Angles $\alpha,\beta,\epsilon,\omega_s,\gamma,\rho$

reception may be anywhere about the sample. The fourth system is hemispherical.

The first system is recommended for specifying laboratory testing. The second and third systems are recommended for study of performance of most retroreflectors. The fourth system is recommended for study of performance of grazing angle retroreflectors.

6.1.1 Fig. 1 illustrates the CIE (goniometer) system $\{\alpha,\beta_1,\beta_2,\epsilon\}$. The four CIE angles follow exactly the motions of the most common design of retroreflectometer. The three motions of the specimen goniometer of Fig. 3 control the angles β_1, β_2 and ϵ, provided the specimen goniometer is situated with respect to the observer goniometer as in Fig. 3. The CIE (goniometer) system has become well established as the basis for photometry of retroreflectors and is preferred by ASTM as the system most likely to produce interlaboratory agreement. However, because entrance angle, unsubscripted β, is considered to be $\pm\beta_1$ in some agencies and considered to be $\pm\beta_2$ in others, there now exist in laboratories in various countries two distinctly different measurement geometries,

termed "coplanar" and "perpendicular", respectively. This creates minor lack of correlation with beaded retroreflectors and serious lack of correlation with prismatic retroreflectors. Fig. 4 illustrates the coplanar geometry. ASTM recommends that β_1 and β_2 both be specified for photometry even if one is zero.

6.1.2 The Intrinsic system $\{\alpha,\beta,\gamma,\omega_s\}$ can be represented (Fig. 5) by a retroreflectometer having a two-axis observer goniometer for setting α and γ, and a two-axis specimen goniometer for setting β and ω_s. An alternative retroreflectometer uses a conventional one-axis observer goniometer for setting α, and a suitably constructed three-axis specimen goniometer for setting β, γ, and ω_s. The angles $\{\alpha,\beta,\gamma,\omega_s\}$ of the Intrinsic system are included in Fig. 2. The performance of prismatic retroreflectors shows clear dependence on the four angles of this system, and that of beaded retroreflectors on the angles α, β, and γ.

NOTE 4—The Intrinsic system is related to the CIE (goniometer) system in that the entrance angle β and the presentation angle γ are together
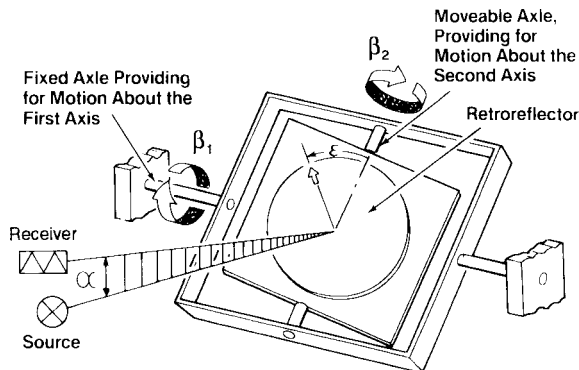

NOTE 1—The sample must be mounted so the retroreflector axis is normal to the plate.
FIG. 3 Positioning a Retroreflector for Testing

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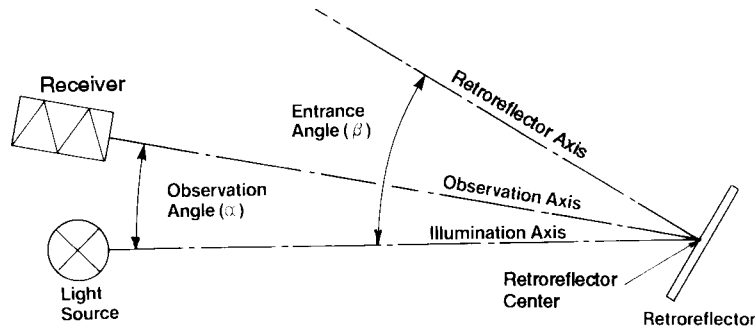
NOTE 1—This figure illustrates a simple test geometry for which the entrance half-plane and the observation half-plane are coplanar. In the CIE (goniometer) system this corresponds to the condition $\beta_2 = 0°$. The entrance angle β and the observation angle α are always positive. The figure does not show the rotation angle ϵ. In the CIE (goniometer) system, β would be labelled β_1 and shown with a single arrow ending at the retroreflector axis, and in this figure β_1 would be positive.

FIG. 4 Coplanar Test Configuration
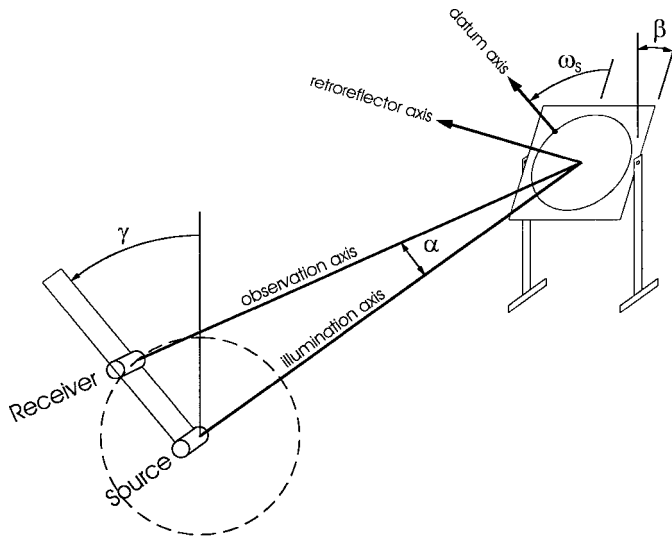
NOTE 1—The retroreflector axis is normal to the face of the sample goniometer. Angles ω_s and γ are shown positive. The receiver track revolves around the illumination axis for setting γ. The β movement is restricted to the direction shown to avoid redundancy.

FIG. 5 Intrinsic System

geometrically equivalent to the pair of entrance angle components β_1 and β_2. The equivalence is given in transformation Eq X1.1, Eq X1.2, Eq X1.4 and Eq X1.6 in the Appendix.

6.1.3 The Application system $\{\alpha, \beta, \epsilon, \omega_s\}$ (Fig. 6) separates the illumination geometry (β, ω_s) from the observation geometry (α, ϵ). The rotational angles ϵ and ω_s are both defined with respect to the sample datum axis. The angles $\{\alpha, \beta, \epsilon, \omega_s\}$ of the Application system are included in Fig. 2. The performance of prismatic retroreflectors shows clear dependence on the four angles of this system. The system is useful for studying the varieties of geometrical demands in road applications. There are no simple goniometers representing this angle system. To use the Application system requires angle transformations into the system of a preferably computerized retroreflectometer.

NOTE 5—In this system, when testing rotationally symmetrical retrore-flectors, it is necessary to specify both angles ω_s and ϵ, because the retroreflectance of such retroreflectors depends on the value of the difference, $\omega_s - \epsilon$.

NOTE 6—The similar system $\{\alpha, \beta, \rho, \omega_s\}$ is useful for optical studies of flux and of diffraction. Transformations between ϵ and ρ are given in X1.4.10.

6.1.4 The RM system $\{a, b, e, d\}$ (Fig. 7) is specialized to nearly flat road markings. The system in popular use is $\{a, e\}$, a two-angle restriction of RM; road markings are almost always measured with $b = 180°$, $d = 0°$. Strictly, all four angles are required, especially for rotationally non-symmetrical road markings.

NOTE 7—The RM system is identical to the European road lighting (RL) system, except that the symbols are altered to avoid confusion with other angles defined in this document. a, b, e, d in RM are called, respectively, α, β, ϵ, δ in RL. Angle ϵ in RL is defined as $90° - \gamma$ in RL.

7. Angle Definition Conventions

7.1 When $\beta_2 = \pm 90°$, the retroreflector axis is perpendicular to the observation half-plane. In this special case the definitions of entrance angle component β_1, second axis, and rotation angle ϵ can no longer be applied. Therefore, by convention, in this special case, β_1 is equal to zero, and the positive part of the second axis is the half-line at the retrore-flector center perpendicular to the illumination axis and in the
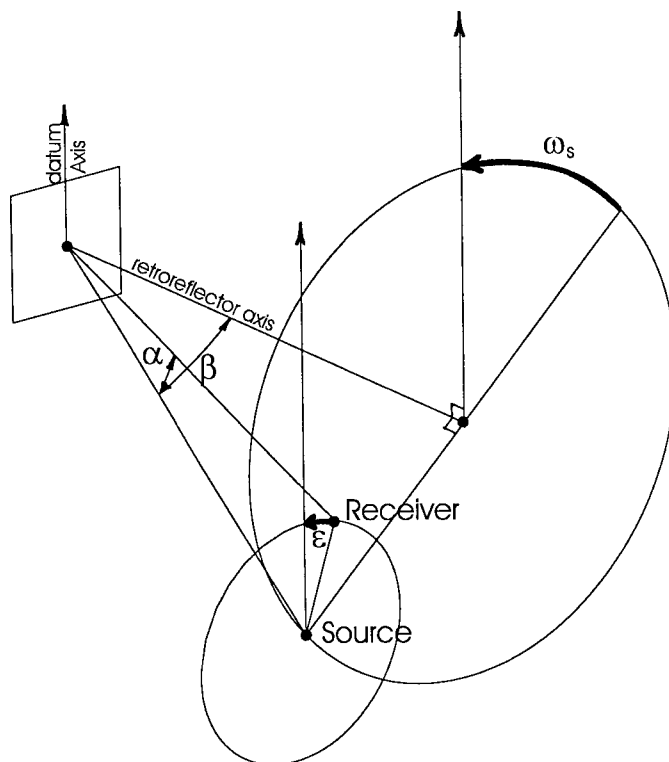
NOTE 1—Angles ω_s and ϵ lie in a plane perpendicular to the retroreflector axis and are shown positive.
FIG. 6 Application System
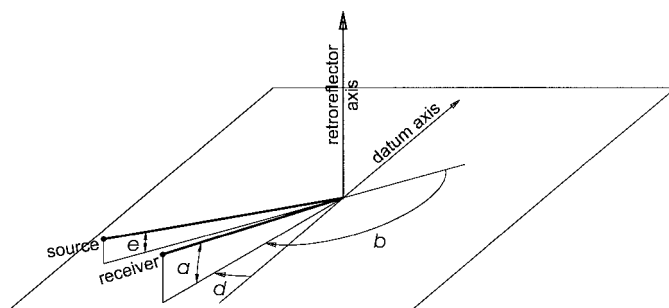
NOTE 1—Angles *d* and *b* are shown positive. Commonly *d*=0°, with *b*=180° and *a*>*e*, the receiver over the source, for testing.
FIG. 7 RM (Road Marking) System

observation half-plane. The secondary definition given in 3.2.38.1 then applies.

7.2 When α=0°, the observation axis and the illumination axis coincide. In this special case, the definition of the observation half-plane, which is used in the definition of several angles, can no longer be applied. Therefore, by convention, β_1=β; β_2=0°; ϵ=ω_s; γ=0°; and ρ= the projection of ω_s into a plane perpendicular to the illumination axis, that is, ρ=tan^{-1}(tanω_s/cosβ).

7.3 When β = 0°, the illumination axis and the retroreflector axis coincide. In this special case, the definition of the entrance half-plane, which is used in the definition of two angles, ω_s and γ, can no longer be applied. Therefore, by convention, ω_s= 0°; γ = −ρ = −ϵ.

8. Specification Conventions

8.1 If the retroreflector has a datum mark and the rotation

angle ϵ is unspecified, it has been a common practice to consider ϵ = 0°. This practice is deprecated by ASTM because the presence of a datum mark indicates that the retroreflector is sensitive to rotation. ASTM recommends that the conditions desired for test be completely specified.

8.2 When the entrance angle β alone is specified without reference to components, it has been a common practice in the United States to consider β_2=0° and β_1=β. Because the use of such conventions results in misunderstandings and conflicting standards, ASTM deprecates the use of this convention and recommends that the conditions desired for test be completely specified. Note in particular that for sign sheeting β_2=0°, β_1=β is a poor representation of the road scenario and may result in misapplication of some materials.

9. Aperture Description Conventions

9.1 Since the efficiencies of retroreflectors are often rapidly

varying functions of the observation angle α and the rho angle ρ, it is usually important to describe the apertures of the source and receiver that are to be used in a measurement. The following conventions for describing apertures are based on the assumptions that: (*1*) the luminance of the source in the direction of the retroreflector is uniform over the source aperture stop, (*2*) the illumination axis passes through the center of the source aperture stop, (*3*) the responsivity of the receiver in the direction of the retroreflector is uniform over the receiver aperture stop, and (*4*) the observation axis passes through the center of the receiver aperture stop.

9.1.1 *Circular Aperture*—The angular size of a circular aperture, either source or receiver, should be described by giving the angle subtended at the retroreflector center by a diameter of the aperture.

9.1.2 *Rectangular Aperture*—If a rectangular aperture, either source or receiver, has one side parallel to the observation half-plane, then its angular size should be described by giving first the angle subtended at the retroreflector center by the side parallel to the observation half-plane and second the angle subtended at retroreflector center by the side perpendicular to the observation half-plane. For example, a 0.1° by 0.2° rectangular aperture has its short side parallel to the observation half-plane.

10. Keywords

10.1 Application system; CIE (goniometer) system; entrance angle; Intrinsic system; observation angle; orientation angle; presentation angle; retroreflection; rotation angle

APPENDIX

(Nonmandatory Information)

X1. TRANSFORMATION TABLES

X1.1 Equations for transformation from the 1959 Brussels CIE coordinate system (α, *E, V, H*) to the CIE (goniometer) system (α, β_1, β_2, ϵ).

Note X1.1—The symbol *E* is used to designate the rotation angle in the 1959 Brussels system to avoid confusion.

$$\alpha = \alpha$$

$$\cos \beta = \cos V \cos H$$

$$\sin \beta_1 = \frac{-\sin V}{(\sin^2 V + \cos^2 V \cos^2 H)^{1/2}}$$

$$\cos \beta_1 = \frac{\cos V \cos H}{(\sin^2 V + \cos^2 V \cos^2 H)^{1/2}}$$

$$\sin \beta_2 = -\sin H \cos V$$

$$\cos \epsilon = \frac{\cos E \cos H + \sin E \sin V \sin H}{(\sin^2 V + \cos^2 V \cos^2 H)^{1/2}}$$

$$\sin \epsilon = \frac{\cos E \sin H \sin V - \sin E \cos H}{(\sin^2 V + \cos^2 V \cos^2 H)^{1/2}}$$

X1.1.1 Special cases: when $V = 0°$ and $H = \pm 90°$

$$\text{then } \beta_2 = \mp 90° \text{ (note sign reversal)}$$

$$\beta_1 = 0°$$

$$\epsilon = -E$$

X1.2 Equations for transformation from CIE (goniometer) system (α, β_1, β_2, ϵ) to the 1959 Brussels CIE coordinate system (α, *E, V, H*).

$$\alpha = \alpha$$

$$\sin V = -\sin \beta_1 \cos \beta_2$$

$$\sin H = \frac{-\sin \beta_2}{(\sin^2 \beta_2 + \cos^2 \beta_1 \cos^2 \beta_2)^{1/2}}$$

$$\cos H = \frac{\cos \beta_1 \cos \beta_2}{(\sin^2 \beta_2 + \cos^2 \beta_1 \cos^2 \beta_2)^{1/2}}$$

$$\cos E = \frac{\sin \epsilon \sin \beta_1 \sin \beta_2 + \cos \epsilon \cos \beta_1}{(\sin^2 \beta_2 + \cos^2 \beta_1 \cos^2 \beta_2)^{1/2}}$$

$$\sin E = \frac{\cos \epsilon \sin \beta_1 \sin \beta_2 - \sin \epsilon \cos \beta_1}{(\sin^2 \beta_2 + \cos^2 \beta_1 \cos^2 \beta_2)^{1/2}}$$

X1.2.1 Special cases: when $\beta_2 = 0°$ and $\beta_1 = \pm 90°$

$$\text{then } H = 0°$$

$$V = \mp 90°$$

$$E = -\epsilon$$

X1.3 In the SAE J594f system, the transformations are the same as in Sections X1.1 and X1.2, with the following conventions:

E	= ϵ_{SAE}(ϵ_{SAE} is rotation angle in SAE J594f)
$\beta_1 > 0$	= *down* SAE J594f angle
$\beta_1 < 0$	= *up* SAE J594f angle
$\beta_2 > 0$	= *right* SAE J594f angle
$\beta_2 < 0$	= *left* SAE J594f angle

X1.4 Equations for transformations between the Intrinsic, Application, CIE (goniometer) and RM systems.

Note X1.2—In these equations $\beta < 90°$ and $-90° < \beta_1 < 90°$.

Note X1.3—The signum function used in these equations is defined as follows: for $x<0$, sgn(x)=–1; for $x>0$, sgn(x)=+1; sgn(0)=0. This agrees with most software, but some define sgn(0)=+1.

X1.4.1 Equations for transformation from Intrinsic system to CIE system are as follows:

$$\beta_1 = \tan^{-1}(\tan \beta \cdot \cos\gamma) \tag{X1.1}$$

$$\beta_2 = \sin^{-1}(\sin\beta \cdot \sin\gamma) \tag{X1.2}$$

$$\epsilon = \omega_s - \tan^{-1}(\tan\gamma \cdot \cos\beta) - 90°\cdot(1 - \text{sgn}(\cos\gamma)) \tag{X1.3}$$

X1.4.2 Equations for transformation from CIE system to Intrinsic system are as follows:

$$\beta = \cos^{-1}(\cos\beta_1 \cdot \cos\beta_2) \tag{X1.4}$$

$$\omega_s = \epsilon + \tan^{-1}\left(\frac{\sin\beta_2}{\tan\beta_1}\right) + 90°\cdot(1 - \text{sgn}(\beta_1)) \tag{X1.5}$$

$$\gamma = \tan^{-1}\left(\frac{\tan\beta_2}{\sin\beta_1}\right) + 90°\cdot(1 - \text{sgn}(\beta_1)) \tag{X1.6}$$

X1.4.2.1 For the special case $\beta_1 = 0° \neq \beta_2$, make $\omega_s = \epsilon + 90°\cdot \text{sgn}(\beta_2)$.

For the special case $\beta_1 = 0° = \beta_2$, make $\omega_s = 0°$.

X1.4.2.2 For the special case $\beta_1 = 0° \neq \beta_2$, make $\gamma = 90° \cdot \text{sgn}(\beta_2)$.

For the special case $\beta_1 = 0° = \beta_2$, make $\gamma = -\epsilon$.

X1.4.3 Equations for transformation from Application system to CIE system are as follows:

$$\beta_1 = \sin^{-1}(\sin\beta \cdot \cos(\omega_s - \epsilon)) \tag{X1.7}$$

$$\beta_2 = \tan^{-1}(\tan\beta \cdot \sin(\omega_s - \epsilon)) \tag{X1.8}$$

X1.4.4 Equations for transformation from CIE system to Application system are as follows:

$$\beta = \cos^{-1}(\cos\beta_1 \cdot \cos\beta_2) \tag{X1.9}$$

$$\omega_s = \epsilon + \tan^{-1}\left(\frac{\sin\beta_2}{\tan\beta_1}\right) + 90°\cdot(1 - \text{sgn}(\beta_1)) \tag{X1.10}$$

X1.4.4.1 For the special case $\beta_1 = 0° \neq \beta_2$, make $\omega_s = \epsilon + 90°\cdot \text{sgn}(\beta_2)$.

For the special case $\beta_1 = 0° = \beta_2$, make $\omega_s = 0°$.

X1.4.5 Equation for transformation from Intrinsic system to Application system is as follows:

$$\epsilon = \omega_s - \tan^{-1}(\tan\gamma \cos\beta) - 90°\cdot(1 - \text{sgn}(\cos\gamma)) \tag{X1.11}$$

X1.4.5.1 For the special cases where $\tan\gamma$ is infinite, make $\epsilon = \omega_s - \gamma$.

X1.4.6 Equation for transformation from Application system to Intrinsic system is as follows:

$$\gamma = \tan^{-1}\left(\frac{\tan(\omega_s - \epsilon)}{\cos\beta}\right) + 90°\cdot(1 - \text{sgn}(\cos(\omega_s - \epsilon))) \tag{X1.12}$$

X1.4.6.1 For the special cases where $\tan(\omega_s - \epsilon)$ is infinite, make $\gamma = \omega_s - \epsilon$.

X1.4.7 Equations for transformation from RM system to Application system are as follows:

$$\alpha = \cos^{-1}(\sin a \sin e - \cos a \cos b \cos e) \tag{X1.13}$$

$$\beta = 90° - e \tag{X1.14}$$

$$\epsilon = d - \tan^{-1}\left(\frac{\tan a \sin b}{\tan e + \tan a \cos b}\right) + 90°\cdot(1 + \text{sgn}(\tan e + \cos a \cos b)) \tag{X1.15}$$

$$\omega_s = d - b + 180° \tag{X1.16}$$

X1.4.8 To transform from RM system to CIE system, first use the equations in X1.4.7 to transform to the Application system, then use the equations in X1.4.3 to transform to the CIE system.

X1.4.9 Equations for transformation from CIE system to RM system are as follows:

$$a = \sin^{-1}(\cos(\beta_1 - \alpha)\cos\beta_2) \tag{X1.17}$$

$$b = 180° + \text{sgn}(\beta_2)\cos^{-1} \left(\frac{\sin^2\beta_2 \cos\beta_1 \cos(\beta_1 - \alpha) + \sin\beta_1 \sin(\beta_1 - \alpha)}{\sqrt{1 - \cos^2\beta_1 \cos^2\beta_2}\sqrt{1 - \cos^2(\beta_1 - \alpha)\cos^2\beta_2}}\right) \tag{X1.18}$$

$$e = \sin^{-1}(\cos\beta_1 \cos\beta_2) \tag{X1.19}$$

$$d = \omega_s + b - 180° \tag{X1.20}$$

X1.4.9.1 To use Eq X1.20 requires first using Eq X1.18 to obtain b and equation Eq X1.5 to obtain ω_s.

X1.4.10 Equations for transformation between rotation angle and rho angle are as follows:

$$\rho = -\tan^{-1}\left(\frac{\tan(\omega_s - \epsilon)}{\cos\beta}\right) + \tan^{-1}\left(\frac{\tan\omega_s}{\cos\beta}\right) + 90°\cdot(\text{sgn}(\cos(\omega_s - \epsilon)) - \text{sgn}(\cos\omega_s)) \tag{X1.21}$$

$$\epsilon = \omega_s - \tan^{-1}\left(\frac{\tan\omega_s \cos\beta - \tan\rho \cos^2\beta}{\cos\beta + \tan\omega_s \tan\rho}\right) + Q \tag{X1.22}$$

Make $Q = 0°$ or $Q = 180°$ so as to produce ϵ in the same quadrant as ρ.