Designation: E 1455 – 97

Standard Practice for
Obtaining Colorimetric Data from a Visual Display Unit Using Tristimulus Colorimeters[1]

INTRODUCTION

This practice provides directions for correcting the results obtained with tristimulus colorimeters when measuring the tristimulus values or chromaticity coordinates of colored displays. Tristimulus colorimeters approximate the CIE color matching functions $\bar{x}$ (λ), $\bar{y}$ (λ), $\bar{z}$ (λ) to make these measurements. The errors generated in measuring colors on a display may be minimized using this practice.

1. Scope

1.1 This practice is intended as an aid for improving the accuracy of colorimetric measurements made with tristimulus colorimeters on visual display units, such as cathode ray tubes (CRTs) and self-luminous flat-panel displays. It explains a useful step in the analysis of colorimetric data that takes advantage of the fact that light from such displays consists of an additive mixture of three primary colored lights. However, it is not a complete specification of how such measurements should be made.

1.2 This practice is limited to display devices and colorimetric instruments that meet linearity criteria as defined in the practice. It is not concerned with effects that might cause measurement bias such as temporal or geometric differences between the instrument being optimized and the instrument used for reference.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

E 284 Terminology of Appearance[2]

E 1336 Test Method for Obtaining Colorimetric Data from a Video Display Unit by Spectroradiometry[2]

E 1341 Practice for Obtaining Spectroradiometric Data from Radiant Sources for Colorimetry[2]

2.2 *ISO/CIE Standard:*

CIE Standard Colorimetric Observers, ISO/CIE 10527: 1991 (E) (International Organization for Standardization, Geneva, 1991)[3]

3. Terminology

3.1 *Definitions:* Unless otherwise stated, definitions of appearance terms in Terminology E 284 are applicable to this practice.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *calibration, n— in reference to a tristimulus colorimeter*, the process performed outside of this practice to adjust the tristimulus colorimeter to provide the best possible results for average or predefined conditions.

3.2.2 *optimization, n— in reference to a tristimulus colorimeter*, the process performed pursuant to this practice to adjust the tristimulus colorimeter or to interpret its readings to provide better results when applied to a particular display device.

3.2.3 *compatible, adj— in reference to a tristimulus colorimeter*, one so designed as to automate the procedure described in this practice.

4. Summary of Practice

4.1 Tristimulus colorimeters comprised of three or four detector channels are, in general, not amenable to accurate calibration that holds for all manner of usage with different illuminated devices and objects. This is because the spectral responsivities of their detector channels do not exactly match the defined Commission Internationale de L'Éclairage (CIE) $\bar{x}$

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[1] This practice is under the jurisdiction of ASTM Committee E-12 on Appearance and is the direct responsibility of Subcommittee E12.06 on Appearance of Displays.

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[2] *Annual Book of ASTM Standards*, Vol 06.01.

[3] Currently available through the U.S. National Committee of the CIE, c/o Mr. Thomas M. Lemons, TLA-Lighting Consultants, Inc., 7 Pond Street, Salem, MA 01970-4819. Also included in *ASTM Standards on Color and Appearance*, Fifth Edition, 1996.

(λ), $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ functions. Factory or subsequent calibration reflects judgments and compromises that may not be readily apparent. Nevertheless, this practice provides guidance on how such a tristimulus colorimeter may be optimized for use with a particular video display device, providing better accuracy with that device than its more general calibration provides. An optimization matrix transforms the instrumental (measured) CIE X, Y, Z values into adjusted X, Y, Z values that are closer to the ideal. This matrix is determined by reference to a colorimeter with higher intrinsic accuracy. The method derives from the fact that the color stimulus functions from display devices are linear combinations of three primary functions and are not entirely arbitrary.

5. Significance and Use

5.1 This practice may be applied when tristimulus colorimeters are used to measure the colors produced on self-luminous video display devices such as CRTs and flat-panel displays, including electroluminescent (EL) panels, field emission displays (FEDs), and back-lit liquid crystal displays (LCDs). This practice is not meant to be a complete description of a procedure to measure the color coordinates of a display. Rather, it provides a method for obtaining more accurate results when certain conditions are met. It may be used by any person engaged in the measurement of color on display devices who has access to the requisite equipment.

5.2 This practice defines a class of tristimulus colorimeters that may be said to be compatible with this practice.

6. Background of Practice

6.1 *Colorimetry*:

6.1.1 Color measurement instruments consist, in general, of means to measure radiometric power as transmitted through a number of bandpass filters. Most commonly, electrical devices are used to measure the filtered light. They may be used with different filters in succession, or multiple devices may be used concurrently. In instruments called spectroradiometers, the radiometric power is measured through a large number (typically 30 to 500) of narrowband filters. (Practice E 1341 describes how a monochromator or polychromator (spectrograph) may be employed to filter and measure light in separate bands on the order of 1-nm wide.) In instruments called tristimulus colorimeters, the radiometric power is measured through three or four wideband filters. These filters may be constructed from dispersive elements (prisms and gratings) or from materials with selective spectral transmission or reflection. The latter may be either uniform or comprised of different patches, in a mosaic pattern, that provide the desired overall effect.

6.1.2 No matter how many filters are used, or in what manner, the goal of the measurement process is to determine tristimulus values X, Y, Z, as defined by ISO in its Standard 10527 and the CIE in its publication No. 15.2 **(1)**.[4] For light with a color stimulus function $\Phi(\lambda)$,

$$X = k\int_{360\,nm}^{830\,nm} \Phi(\lambda)\bar{x}(\lambda)d\lambda \tag{1}$$

$$Y = k\int_{360\,nm}^{830\,nm} \Phi(\lambda)\bar{y}(\lambda)d\lambda \tag{1}$$

$$Z = k\int_{360\,nm}^{830\,nm} \Phi(\lambda)\bar{z}(\lambda)d\lambda \tag{1}$$

where:

k is 683 lm/W for emissive devices, such as displays, and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are color-matching functions. While the standard definition of X, Y, Z requires the use of the CIE 1931 2° color-matching functions, the mathematics described in this practice would also be applicable to any other set of color-matching functions, such as the CIE 1964 10° functions.

6.1.3 In practice, color measurement instruments compute X, Y, Z by the summation of the signals as measured through the various filters, each signal being multiplied by an appropriate calibration factor. In matrix notation:

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} C_{X1} & C_{X2} & C_{\boldsymbol{X3}} & \cdots & C_{Xf} \\ C_{Y1} & C_{Y2} & C_{Y3} & \cdots & C_{\mathbf{Y}f} \\ C_{Z1} & C_{Z2} & C_{Z3} & \cdots & C_{Zf} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_f \end{bmatrix} \tag{2}$$

where:

F_1, F_2, F_3, through F_f are the electrical signals from the f filtered detectors and the C_{ij} are calibration coefficients. X_m, Y_m, Z_m have subscripts to indicate that they are measured values rather than ideal ones.

6.1.4 In this practice, we presume that the color measuring instrument is linear: that each signal F_a is strictly proportional to the received optical power, that any zero-offset (background in darkness) is removed, that the proportionality for signal F_a is not affected by the value of signal F_b, and in the case of closely packed detectors (such as charge-coupled device (CCD) detector elements) no signal F_a spills over and affects signal F_b as it approaches saturation. These presumptions are amenable to experimental verification using methods beyond the scope of this practice **(2)**.

6.1.5 The values of the matrix elements C_{ij} may be determined using criteria that depends on the design and intended application of the instrument. The full extent of this subject is beyond the scope of this practice. However, in general, for spectroradiometers ($f \approx 30$ to 500), C_{Xj} reflects the tabulated value of $\bar{x}(\lambda)$ near the center wavelength of Filter j as well as the spectral responsivity of the corresponding detector channel. (Likewise, C_{Yj} and C_{Zj} reflect $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$, respectively.) For tristimulus colorimeters, the choice of C_{ij} is discussed further, below. As a general matter, the instrument designer should choose passbands and matrix elements that balance accuracy, sensitivity, and other design requirements.

6.1.6 Tristimulus colorimeters are generally designed with filters that are intended to match the spectral responsivities of their detector channels to the CIE $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ functions. For such an instrument,

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} C_{X1} & 0 & 0 \\ 0 & C_{Y2} & 0 \\ 0 & 0 & C_{Z3} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{3}$$

where:

the non-zero C_{ij} matrix elements represent adjustable gains of the detector channels. However, the $\bar{x}(\lambda)$ function has two distinct lobes. This may be dealt with by splitting $\bar{x}(\lambda)$ into $\bar{x}_{\text{short}}(\lambda)$ and $\bar{x}_{\text{long}}(\lambda)$, each with a separate filter (F_1 and F_2,

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[4] Boldface numbers in parentheses refer to items in the list of References at the end of this practice.

respectively). For such an instrument,

$$\begin{bmatrix} X_{\mathrm{m}} \\ Y_{\mathrm{m}} \\ Z_{\mathrm{m}} \end{bmatrix} = \begin{bmatrix} C_{X1} & C_{X2} & 0 & 0 \\ 0 & 0 & C_{Y3} & 0 \\ 0 & 0 & 0 & C_{Z4} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \tag{4}$$

Alternatively, the $\bar{z}(\lambda)$ function may serve the role of $\bar{x}_{\mathrm{short}}(\lambda)$ since they have a similar shape,

$$\begin{bmatrix} X_{\mathrm{m}} \\ Y_{\mathrm{m}} \\ Z_{\mathrm{m}} \end{bmatrix} = \begin{bmatrix} C_{X1} & 0 & C_{X3} \\ 0 & C_{Y2} & 0 \\ 0 & 0 & C_{Z3} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{5}$$

In all of these cases, it is difficult to realize an exact match between the CIE color-matching functions and the actual spectral responsivities of the corresponding detector channels. This means that no choice of C_{ij} will provide perfect calibration for all applications of the instrument. The criteria for setting the C_{ij} might not be well documented for a particular instrument.

6.1.7 It is generally believed that spectroradiometers, with their many detector channels, may be calibrated to yield superior measurements of X, Y, Z for diverse applications. Nevertheless, the relative simplicity of tristimulus colorimeters and their commensurately lower cost have made them popular where the highest accuracy is not required.

6.2 *Self-Luminous Displays*:

6.2.1 A self-luminous display, such as a CRT, an electroluminescent (EL) panel, a field emission display (FED), or a back-lit liquid crystal display (LCD) generates colored light by the proportional superposition (addition) of primary colored lights $\Phi_{\mathrm{r}}(\lambda)$, $\Phi_{\mathrm{g}}(\lambda)$, $\Phi_{\mathrm{b}}(\lambda)$. The subscripts represent red, green, and blue, the primary colors of an additive set. An arbitrarily colored patch on the visual display has one and only one color stimulus function $\Phi(\lambda)$,

$$\Phi(\lambda) = a\Phi_r(\lambda) + b\Phi_g(\lambda) + c\Phi_b(\lambda) \tag{6}$$

where a, b, c are coefficients that are determined by the display electronics.

6.2.2 The display electronics vary a, b, c over the face of the display in order to generate a colored image. For this practice, we presume that the display electronics may be set to make a, b, c uniform (perhaps after averaging nonobvious fine-structure) over a sufficient area of the display to permit measurements to be made on that area.

6.2.3 It is a requirement for the applicability of this practice that the display device behaves as stated in Eq 6. This practice does not represent that any particular display device will act as predicted by Eq 6, though those within the mentioned classes of devices might do so. The procedure for experimental verification of this property for a specific display device is beyond the scope of this practice **(3)**.

6.3 *Colorimetric Measurement of Displays*:

6.3.1 Each of the primary color stimulus functions $\Phi_{\mathrm{r}}(\lambda)$, $\Phi_{\mathrm{g}}(\lambda)$, $\Phi_{\mathrm{b}}(\lambda)$ stimulates responses in the f detector channels that may be represented by a vector $\boldsymbol{F}$ (that is, $\boldsymbol{F}_{\mathrm{r}}$, $\boldsymbol{F}_{\mathrm{g}}$, $\boldsymbol{F}_{\mathrm{b}}$). Given their construction, these vectors are linearly independent. (None of the three can be expressed as a linear combination of the other two.) While $\boldsymbol{F}$ is an element of an f-dimensional vector space, it is clear that only a three-dimensional subspace is spanned by the $\boldsymbol{F}$'s of all possible color stimulus functions following Eq 6. Further, the mapping of $\boldsymbol{F}$ into (X_{m}, Y_{m}, Z_{m}) space by Eq 2 remains three dimensional. In other words, there is a one-to-one mapping of the vector (a, b, c) onto (X, Y, Z) by application of Eq 1; and, for a particular instrument with a fixed calibration matrix C, there is also a one-to-one mapping of the vector (a, b, c) onto (X_{m}, Y_{m}, Z_{m}). From this we deduce that a matrix R exists that can be used to translate (X_{m}, Y_{m}, Z_{m}) values into actual (X, Y, Z) values.

6.3.2 A colorimeter that takes advantage of this fact must provide means for implementing the matrix R. That is, all f filtered detector signals should contribute linearly toward the computation of each output, X_{m}, Y_{m}, Z_{m}, instead of using different detectors for each output. This idea was reported as long ago as 1973 by Wagner **(4)**, and it has been expanded upon and rediscovered by others since then **(5-9)**.

6.3.3 On the basis of this property, a tristimulus colorimeter can be optimized for use on a self-luminous display by the proper derivation of a matrix R for that display. We proceed on the assumptions that the components are sufficiently stable, and that similarly built displays have similar enough spectral primaries to make a derivation of R worthwhile. However, these assumptions should be quantified before accuracy claims are made in any specific situation.

6.3.4 On the basis of this property, a tristimulus colorimeter designed for use with displays need not produce signals $\boldsymbol{F}$ that are close to CIE tristimulus values. Signal/noise may be improved by matching the spectral responsivities of the filtered detectors to the emission spectra of the primary colors. In such designs, it is especially important to use a matrix R that is specific to the particular $\Phi_{\mathrm{r}}(\lambda)$, $\Phi_{\mathrm{g}}(\lambda)$, $\Phi_{\mathrm{b}}(\lambda)$.

7. Optimization

7.1 *General*:

7.1.1 Given the existence of a matrix R, how is it determined? Experimentally, the problem is one of comparing the data X, Y, Z from a reference colorimeter with the data X_{m}, Y_{m}, Z_{m} from the colorimeter being optimized, for a number of color samples at different display settings. From these data, R is calculated.

7.1.2 This practice is not directly concerned with the absolute accuracy of the measurements. It concerns the transfer of calibration from a reference instrument to another instrument, regardless of the absolute accuracy of the reference.

7.2 *Noiseless Data*:

7.2.1 For clarity, we first consider the case in which the measuring instruments are free of noise. We define vectors $\boldsymbol{n}$ and $\boldsymbol{m}$ as:

$$n = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad ; m = \begin{bmatrix} X_{\mathrm{m}} \\ Y_{\mathrm{m}} \\ Z_{\mathrm{m}} \end{bmatrix} \tag{7}$$

where:

each is an element of its corresponding vector space, and both derive from the same (a, b, c) setting on a display. The matrix R maps between them:

$$n = R\,m\,. \tag{8}$$

7.2.2 This relationship may be stated for more than one such pairs of vectors (for multiple display settings) at the same time:

$$N = R\,M \tag{9}$$

where:

$$N = \begin{bmatrix} X_1\, X_2\, X_3 \ldots X_i \\ Y_1\, Y_2\, Y_3 \ldots Y_i \\ Z_1\, Z_2\, Z_3 \ldots Z_i \end{bmatrix} \qquad (10)$$

and

$$M = \begin{bmatrix} X_{m1}\, X_{m2}\, X_{m3} \ldots X_{mi} \\ Y_{m1}\, Y_{m2}\, Y_{m3} \ldots Y_{mi} \\ Z_{m1}\, Z_{m2}\, Z_{m3} \ldots Z_{mi} \end{bmatrix} \qquad (11)$$

7.2.3 When matrices N and M have exactly three columns, and when their columns are linearly independent, R may be easily determined:

$$R = N\, M^{-1} \qquad (12)$$

The determination of matrix R in this case requires the reference values and the test colorimeter measurements of exactly three distinct colors on the display.

7.3 *Real-World Transformations of X, Y, Z*:

7.3.1 In practice, neither measurements of $\boldsymbol{n}$ nor $\boldsymbol{m}$ are made with perfect accuracy. Noise affects the measurements, the linearity presumptions in 6.1.4 and 6.2.3 may not be perfectly true, and there may be other unexpected systematic effects that affect the data. Therefore, it is prudent to determine R by using more than three color samples and by using statistical methods.

7.3.2 For a given R_0 and for a pair of related $\boldsymbol{n}$ and $\boldsymbol{m}$ vectors:

$$n = R_0\, m + v \qquad (13)$$

where:

$\boldsymbol{n}$ and $\boldsymbol{m}$ are as in Eq 7, and $\boldsymbol{v}$ is the difference vector between the reference tristimulus values $\boldsymbol{n}$ and those computed by the use of Eq 8. For several different colors, Eq 13 can be expressed as:

$$N = R_0\, M + V \qquad (14)$$

where:

N is as in Eq 10,

M is as in Eq 11,

and:

$$V = \begin{bmatrix} v_{11}\, v_{12}\, v_{13} \ldots v_{1i} \\ v_{21}\, v_{22}\, v_{23} \ldots v_{2i} \\ v_{31}\, v_{32}\, v_{33} \ldots v_{3i} \end{bmatrix} \qquad (15)$$

7.3.3 When many measurements $\boldsymbol{n}$ and $\boldsymbol{m}$ are available, the optimal R_0, hereafter called R', may be determined by minimization of the sum of the squares of the elements of V with respect to all nine elements of R'. This is the statement of least-squares fitting when applied to the vector space of R. It follows that:

$$R' = N\, [(M\, M^T)^{-1}\, M]^T \qquad (16)$$

where:

the superscript T implies matrix transposition, and where we presume that the statistical uncertainty (as opposed to the measurement standard uncertainty) of all X, Y, Z and X_m, Y_m, Z_m are the same, that is, each is known to ± the same value. As with all least-squared fitting procedures, the results will be affected by the portions of R space that are sampled the most and by the presumption of identical statistical weights (uncertainties) for each data sample. When R' is used for R_0 in Eq 14, the residuals V may be examined for trends that might indicate systematic effects.

7.4 *Real-World Transformations of Y, x, y*:

7.4.1 Eq 16 is the principal result of Wagner **(4)**. However, there is an important situation in which all of X, Y, Z and X_m, Y_m, Z_m are not known with equal uncertainties. Many commercial instruments report Y, x, y instead of X, Y, Z,

where:

$$x = \frac{X}{X + Y + Z};\; y = \frac{Y}{X + Y + Z}\,. \qquad (17)$$

(We have dropped the m subscripts in Eq 17 for clarity.)

7.4.2 In order for this practice to apply, the instrument must transform X, Y, Z to Y, x, y accurately (internally, in the colorimeter). Instruments with analog circuits that only approximate Eq 17 are not covered.

7.4.3 One may not simply convert Y, x, y to X, Y, Z and proceed as in 7.3. Instruments typically report x, y to a limited precision. Transformations of Y, x, y back to X, Y, Z do not preserve the statistical uncertainties needed for the data fitting.

7.4.4 Calculation of R' for this case cannot be done as compactly as it was in Eq 16. Instead, we must calculate it a row at a time. Expressing R' as its individual matrix elements, and showing its rows,

$$R' = \begin{bmatrix} R'_{XX}\, R'_{XY}\, R'_{XZ} \\ R'_{YX}\, R'_{YY}\, R'_{YZ} \\ R'_{ZX}\, R'_{ZY}\, R'_{ZZ} \end{bmatrix} \qquad (18)$$

7.4.5 Calculation of the middle row is much the same as in 7.3.

$$(R'_{YX}\, R'_{YY}\, R'_{YZ}) = N_Y\, [(M_Y\, M_Y^T)^{-1}\, M_Y]^T, \qquad (19)$$

where:

$$N_Y = (Y_1\, Y_2\, Y_3 \ldots Y_i)\,, \qquad (20)$$

and

$$M_Y = \begin{bmatrix} X_{m1}\, X_{m2}\, X_{m3} \ldots X_{mi} \\ Y_{m1}\, Y_{m2}\, Y_{m3} \ldots Y_{mi} \\ Z_{m1}\, Z_{m2}\, Z_{m3} \ldots Z_{mi} \end{bmatrix} \quad (\textbf{as in Eq 11}), \qquad (21)$$

with

$$X_{mi} = Y_{mi}\frac{x_{mi}}{y_{mi}},\;\; Z_{mi} = Y_{mi}\frac{(1 - x_{mi} - y_{mi})}{y_{mi}}\,. \qquad (22)$$

7.4.6 It is useful to compute

$$(Y'_1\, Y'_2\, Y'_3 \ldots Y'_i) = (R'_{YX}\, R'_{YY}\, R'_{YZ})\, M_Y, \qquad (23)$$

where:

Y' are the best-fit luminances of the sample display settings. In some circumstances it is better to use Y' rather than Y values in computing the remaining elements of R'. For example, in the process of computing x, y from X, Y, Z, an instrument reduces the effects of display flicker in the data. Use of Y' maintains this noise reduction in the subsequent data fits. On the other hand, if some aspect of the display is varying slowly in time, or actually varies based on the display setting, then it may be better to use Y rather than Y' values in order to make the best comparisons between individual readings of the two instruments. Although the equations in the following sections are written with Y' rather than Y, both versions of the formulae should be considered.

7.4.7 With statistical weighting,

$$N_X = \left(\frac{Y'_1 \frac{x_1}{y_1}}{\sigma_{X1}} \frac{Y'_2 \frac{x_2}{y_2}}{\sigma_{X2}} \frac{Y'_3 \frac{x_3}{y_3}}{\sigma_{X3}} \ldots \frac{Y'_i \frac{x_i}{y_i}}{\sigma_{Xi}} \right) \tag{24}$$

and

$$M_X = \begin{bmatrix} \frac{X_{\mathrm{m}1}}{\sigma_{\mathrm{X}1}} \frac{X_{\mathrm{m}2}}{\sigma_{\mathrm{X}2}} \frac{X_{\mathrm{m}3}}{\sigma_{\mathrm{X}3}} \ldots \frac{X_{\mathrm{m}i}}{\sigma_{\mathrm{X}i}} \\ \frac{Y_{\mathrm{m}1}}{\sigma_{\mathrm{X}1}} \frac{Y_{\mathrm{m}2}}{\sigma_{\mathrm{X}2}} \frac{Y_{\mathrm{m}3}}{\sigma_{\mathrm{X}3}} \ldots \frac{Y_{\mathrm{m}i}}{\sigma_{\mathrm{X}i}} \\ \frac{Z_{\mathrm{m}1}}{\sigma_{\mathrm{X}1}} \frac{Z_{\mathrm{m}2}}{\sigma_{\mathrm{X}2}} \frac{Z_{\mathrm{m}3}}{\sigma_{\mathrm{X}3}} \ldots \frac{Z_{\mathrm{m}i}}{\sigma_{\mathrm{X}i}} \end{bmatrix} \tag{25}$$

Ideally we would know the actual statistical uncertainties $\sigma_{\mathrm{x}i}$, but we consider the situation in which the two instruments (reference and test) both report *x*, *y* to ± a constant, such as 0.001. In this case, the measurement precision is limited by the truncation and rounding to three digits. Following the standard rules for propagation of uncertainties, the relative statistical uncertainties are

$$\sigma_{Xi} = Y'_i \frac{x_i}{y_i} \sqrt{\left(\frac{\Delta x}{x_i}\right)^2 + \left(\frac{\Delta y}{y_i}\right)^2 + \left(\frac{\Delta Y}{Y'_i}\right)^2}, \tag{26}$$

where Δx, Δy (and later, Δz) are taken as 0.001 (for the sake of example, as these are roughly the standard uncertainties that might be expected in this situation), and ΔY is taken as the experimental standard deviation of the set of values $\{(Y'_i - Y_i)\}$. From this, we can calculate as follows:

$$(R'_{XX} R'_{XY} R'_{XZ}) = N_X[(M_X M_X^T)^{-1} M_X]^T, \tag{27}$$

7.4.8 Similarly,

$$N_Z = \left(\frac{Y'_1 \frac{z_1}{y_1}}{\sigma_{Z1}} \frac{Y'_2 \frac{z_2}{y_2}}{\sigma_{Z2}} \frac{Y'_3 \frac{z_3}{y_3}}{\sigma_{Z3}} \ldots \frac{Y'_i \frac{z_i}{y_i}}{\sigma_{Zi}} \right) \tag{28}$$

and

$$M_Z = \begin{bmatrix} \frac{X_{\mathrm{m}1}}{\sigma_{\mathrm{Z}1}} \frac{X_{\mathrm{m}2}}{\sigma_{\mathrm{Z}2}} \frac{X_{\mathrm{m}3}}{\sigma_{\mathrm{Z}3}} \ldots \frac{X_{\mathrm{m}i}}{\sigma_{\mathrm{Z}i}} \\ \frac{Y_{\mathrm{m}1}}{\sigma_{\mathrm{Z}1}} \frac{Y_{\mathrm{m}2}}{\sigma_{\mathrm{Z}2}} \frac{Y_{\mathrm{m}3}}{\sigma_{\mathrm{Z}3}} \ldots \frac{Y_{\mathrm{m}i}}{\sigma_{\mathrm{Z}i}} \\ \frac{Z_{\mathrm{m}1}}{\sigma_{\mathrm{Z}1}} \frac{Z_{\mathrm{m}2}}{\sigma_{\mathrm{Z}2}} \frac{Z_{\mathrm{m}3}}{\sigma_{\mathrm{Z}3}} \ldots \frac{Z_{\mathrm{m}i}}{\sigma_{\mathrm{Z}i}} \end{bmatrix} \tag{29}$$

where:

$$z_i = 1 - x_i - y_i, \sigma_{Zi} = Y'_i \frac{z_i}{y_i} \sqrt{\left(\frac{\Delta z}{z_i}\right)^2 + \left(\frac{\Delta y}{y_i}\right)^2 + \left(\frac{\Delta Y}{Y'_i}\right)^2}, \tag{30}$$

and

$$(R'_{ZX} R'_{ZY} R'_{ZZ}) = N_Z [(M_Z M_Z^T)^{-1} M_Z]^T, \tag{31}$$

8. Application

8.1 *Manual Optimization*:

8.1.1 The optimization of a tristimulus colorimeter requires knowing the true CIE tristimulus values *X, Y, Z* of at least seven colors that can be presented on the display, three of which must be linearly independent. It is recommended that these colors be the brightest red, green, blue, yellow, cyan, magenta, and white that the display normally produces. Their tristimulus values must be determined by a well-trusted instrument, such as a spectroradiometer being used according to Test Method E 1336, or a tristimulus colorimeter with a much closer match to the CIE $\bar{x}$ (λ), $\bar{y}$ (λ), $\bar{z}$ (λ) functions than the colorimeter being optimized. These values are entered into matrix *N*, following Eq 10.

8.1.2 Measurements are made of these colors with the tristimulus colorimeter being optimized. These values are entered into matrix *M*, following Eq 11.

8.1.3 Matrix *R′* is computed, following Eq 16.

8.1.4 If only *X, Y, Z* data are available, rather than *X, Y, Z* data, then the analogous steps in 7.4 are followed instead.

8.1.5 Subsequent measurements made by the same tristimulus colorimeter on the same display are converted using Eq 8, where *R′* is used for *R*. The resultant ***n*** vector is taken as the optimized (adjusted) result of the measurement.

8.2 *Automatic Optimization*:

8.2.1 All tristimulus colorimeters have, in some form, an internal calibration matrix *C* that relates detector signals to tristimulus values, as described in Eq 2. *C* may be required to reduce the dimensionality of the instrument from *f* to 3.

8.2.2 A tristimulus colorimeter may, by design, incorporate a provision to transform a tentative determination of *X, Y, Z* (that is, X_{m}, Y_{m}, Z_{m}) to a more accurate determination of *X, Y, Z* by the application of Eq 8. Such a colorimeter contains, in some form, an embodiment of matrix *R*. This matrix is distinguished from the *C* matrices shown in Eq 3-5 by the fact that it contains, in general, no zero elements. That is, all three inputs contribute to the determination of all three tristimulus values, with nine degrees of calibration freedom. *C* and *R* may be implemented jointly.

8.2.3 A tristimulus colorimeter may be described as compatible with this standard practice if and only if (*1*) it includes an implementation of matrix *R*, and (*2*) the user of the colorimeter has means to set the nine *R* matrix elements to allow continued, automatic application of Eq 8 after the *R′* matrix has been determined. A compatible colorimeter may also provide means to compute *R′* by allowing the user to enter the true *X, Y, Z* of at least seven color samples. These implementations may be either locally dedicated to the colorimeter or installed in a computer interfaced to the colorimeter.

9. Precision and Bias

9.1 The typical precision and bias of tristimulus value measurements made using this practice have yet to be determined. However, it is expected that the limits of precision and the origins of bias in such measurements will arise from causes outside of the scope of this practice, such as those discussed as follows:

9.1.1 This practice transfers the calibration from a well-trusted spectroradiometer or a colorimeter to another instrument, a tristimulus colorimeter. No matter how well the transfer is made, the bias of the final measurements will reflect the precision and bias of the reference device.

9.1.2 This practice is limited to discussion of the spectral characteristics of the instruments and does not consider any biases that might arise from the geometry of collection of radiant flux by either the tristimulus colorimeter or the reference device. For example, the spectral radiance of an active matrix liquid crystal (AMLCD) display is highly directional. Such a display could not be used to transfer calibration between instruments with different numerical apertures. Also, displays exhibit spatial nonuniformities in their radiance. The inability of dissimilar instruments to measure the same area of a display can limit the accuracy of this technique.

9.1.3 This practice does not consider the temporal response characteristics of either the tristimulus colorimeter or the reference device. The user is reminded that many types of displays are scanned, and the resultant flicker may impart a bias, particularly if the colorimeter and the reference device are dissimilar.

9.1.4 This practice presumes linear instrumental and display behavior, as described in 6.1.4 and 6.2.3. The measurements may be biased if these presumptions are incorrect.

9.1.5 This practice does not consider biases that might arise due to the environment in which it is performed. Specifically, ambient light may affect the measurements if they are not made in a dark room. Changes in ambient temperature may affect the performance of either the tristimulus colorimeter or the reference device.

9.1.6 The selection of the filter passbands (6.1.1) and the selection of the C_{ij} matrix elements when $f > 3$ (6.1.3-6.1.6) are topics beyond the scope of this practice. However, these choices affect the precision of the technique by emphasizing or deemphasizing statistically significant differences between the detector channel signals that correlate to the differences in the tristimulus values of the light source.

9.2 This practice is intended to help minimize the errors in the determination of the tristimulus values X, Y, Z. No representation is made that this practice minimizes errors in the determination of chromaticity coordinates x, y or u, v, perceptible color differences, or any other values computed from X, Y, Z.

10. Keywords

10.1 calibration (tristimulus colorimeters); cathode ray tubes (CRTs); chromaticity; CIE color-scale system; CIE tristimulus values; color; data analysis; flat-panel displays; instrumental measurement (color/light); liquid crystal displays (LCDs); luminance; matrix transformation; self-luminous displays; tristimulus colorimeters; tristimulus values; visual/video display units (VDUs)

APPENDIX

(Nonmandatory Information)

X1. NUMERICAL EXAMPLE

X1.1 An example of the optimization procedure is given here for the case in which the chromaticity coordinates (rather than the tristimulus values) are known.

X1.2 A CRT is made to display eight elementary colors (more than the minimum required number of seven). Each color is measured by two colorimeters, a reference instrument and the instrument being optimized (the target). The data are given in Table X1.1.

X1.3 The reference luminance (Y) values are arranged in the form of a 1×8 N_Y matrix, as in Eq 20. The target data are converted from Y, x, y to X, Y, Z format (X_m, Y_m, Z_m) using Eq 22, and matrix M_Y is constructed with eight columns as in Eq 21. The middle row of the R' matrix (Eq 18) is computed using Eq 19.

X1.4 Using $M_{Y'}$ and the results of the previous step, the best-fit Y are computed using Eq 23. These results are also shown in Table X1.1. The rms difference between the corresponding reference Y and Y' values is 0.125, which is taken to be ΔY in the following steps.

TABLE X1.1 Luminance and Chromaticity Coordinates of Eight Sample Colors

Elementary Color	Reference			Target			Y'
	Y	x	y	Y	x	y	
Red	12.25	0.617	0.351	11.50	0.620	0.350	12.40
Green	38.45	0.294	0.604	36.25	0.293	0.605	38.47
Yellow	50.85	0.409	0.516	47.53	0.413	0.514	50.64
Blue	6.43	0.150	0.075	5.11	0.153	0.070	6.32
Magenta	18.65	0.286	0.154	16.43	0.293	0.153	18.55
Cyan	44.70	0.211	0.301	41.25	0.213	0.299	44.74
White	56.75	0.289	0.311	52.55	0.293	0.309	56.92
Gray	11.90	0.297	0.323	10.95	0.302	0.320	11.84

X1.5 For each of the eight samples, an uncertaintyσ_{Xi} is computed using Eq 26. This computation uses the best-fit Y' determined in X1.4, the reference x, y from Table X1.1, ΔY as determined in X1.4, and 0.001 for Δx and Δy. The values 0.001 are adopted because the instruments in this example report x, y to ±0.001.

X1.6 Matrix N_X(Eq 24), one row of eight columns, is constructed using the reference x, y data in Table X1.1 and the results of the previous two steps. Matrix M_X (Eq 24), three rows of eight columns, is constructed using the uncertainties computed in X1.5 and the X_m, Y_m, Z_m computed in X1.4. The top row of the R matrix (Eq 18) is computed using Eq 27.

X1.7 For each of the eight samples, an uncertainty σ_{Zi} is computed using Eq 30. This computation parallels that in X1.5.

X1.8 Matrices N_Z (Eq 28) and M_Z (Eq 29) are constructed analogous to the way N_X and M_X were in X1.6. The bottom row of the R' matrix (Eq 18) is computed using Eq 31.

X1.9 In this example,

$$R' = \begin{bmatrix} 1.0536 & 0.0007 & 0.0088 \\ 0.0144 & 1.0519 & 0.0138 \\ 0.0081 & -0.0080 & 1.0861 \end{bmatrix} \qquad (X1.1)$$

X1.10 R' is applied to all subsequent measurements using the target instrument on the same CRT (or perhaps type of

CRT) as used for the data in Table X1.1. *Y*, *x*, *y* data from the target instrument, shown in Table X1.2, are converted to *X*, *Y*, *Z* format, multiplied by R' as in Eq 8, and converted back to *Y*, *x*, *y* format. These results are shown in Table X1.2 as the Optimized Target. In this example, the results may be compared to the corresponding readings from the reference instrument, although normally the reference instrument would no longer be used after R' is determined.

TABLE X1.2 Application of Optimization Transform to Raw Data

Reference			Target			Optimized Target		
Y	*x*	*y*	*Y*	*x*	*y*	*Y*	*x*	*y*
36.10	0.236	0.258	33.00	0.239	0.256	36.05	0.236	0.258
17.70	0.551	0.404	16.55	0.556	0.401	17.76	0.551	0.404
5.82	0.314	0.573	5.49	0.315	0.576	5.83	0.314	0.576
28.50	0.256	0.216	25.85	0.262	0.215	28.51	0.258	0.218
16.90	0.331	0.186	15.10	0.339	0.185	16.81	0.333	0.190
12.80	0.608	0.357	11.95	0.613	0.356	12.88	0.606	0.360
12.30	0.290	0.236	11.10	0.298	0.234	12.18	0.294	0.237
40.20	0.318	0.585	37.70	0.318	0.585	40.04	0.317	0.586
5.59	0.295	0.587	5.27	0.294	0.591	5.60	0.293	0.591
10.10	0.481	0.454	9.35	0.488	0.450	10.00	0.484	0.452

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